Polyominoes

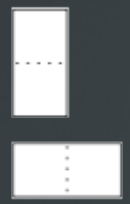
Andrés Jaimes

Definition

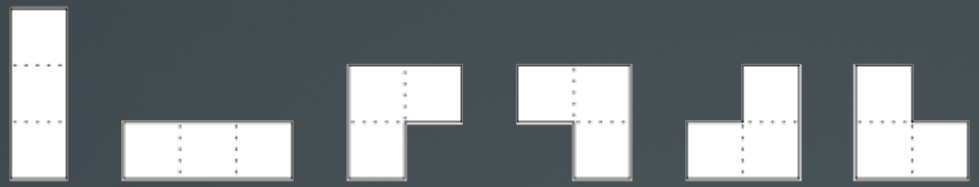
- Connected set of *n* squares through edges only

Definition

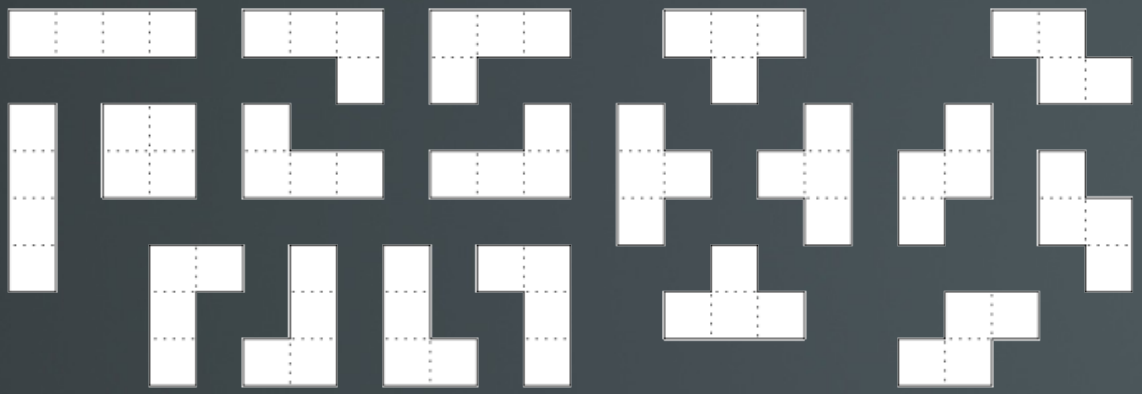
- Fixed: Distinct if they have different shapes or orientations.



Dominoes



Triominoes



Tetrominoes

Definition

- Free: Equal if they can be obtained by rotating or reflecting fixed polyominoes.

Definition

A(n) denotes the number of polyominoes of size *n*.

- Fixed:
  - A(2) = 2
  - A(3) = 6
  - A(4) = 19
  - A(5) = 63
  - A(6) = 216
  - A(12) = 505,861
- Free:
  - A(2) = 1
  - A(3) = 2
  - A(4) = 5
  - A(5) = 12
  - A(6) = 35
  - A(12) = 63,600

Interesting Facts

- No analytic formula for $A(n)$ is known.
- Iwan Jensen has enumerated the fixed polyominoes up to $n = 56$: $A(56)$ is approximately 6.915×10^{31}
- Free polyominoes have been enumerated up to $n=28$

Algorithms for enumeration

- Inductive
- Redelmeier (Hugh Redelmeier)
- Transfer Matrix (by Andrew Conway, improved by Iwan Jensen in 2003, and optimized by Donald Knuth)
- The Twisted Cylinder (Ares Ribó Mor, 2005)

Redelmeier's

- Enumerate fixed
- Discard rotated, translated and reflected fixed polyominoes to get the free ones

Redelmeier's

- Define a standard form: First cell at the origin
- Depth-first traversal
- Child = Parent + One adjacent cell
- Untried set: adjacent points not tried by ancestors or older brothers

Redelmeier's

- Repeat until untried set is exhausted
  - Remove an arbitrary element from the untried set
  - Place a cell at this point
  - Count this new polyomino
  - If the size is less than P:
    - (a) Add new neighbors to the untried set.
    - (b) Call this algorithm recursively with the new parent being the current polyomino, and the new untried set being a copy of the current one.
    - (c) Remove the new neighbors from the untried set.
  - 5, Remove newest cell

References

- Redelmeier, D. H. “Counting Polyominoes: Yet Another Attack”. Discrete Math. 36, 191-203, 1981.
- “Polyomino”. http://en.wikipedia.org/wiki/Polyomino
- Weisstein, Eric W. “Polyomino”. http://mathworld.wolfram.com/Polyomino.html
- Ares Ribó Mor. “Counting Polyominoes on Twisted Cylinders”. http://www.diss.fu-berlin.de/2006/353/chapter7_polyominoes.pdf
- Knuth, Donald E. “Polynum/Polyslave”. http://www-cs-staff.stanford.edu/~uno/programs.html
- Garrido, Javier. “Teoría de Poliominós”. http://www.dma.fi.upm.es/docencia/primerciclo/matrecreativa/juegos/poliominos/inicio.html